

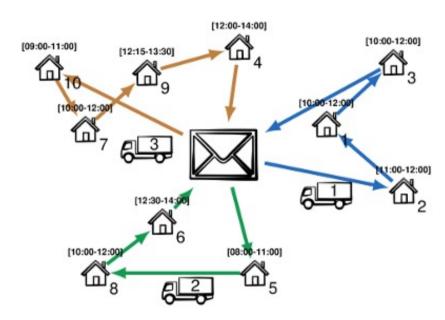


Genetic Programming Hyper-heuristics for Combinatorial Optimisation

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Combinatorial Optimisation

- Important (many real-world applications)
- Hard to solve (usually NP-hard)
- Examples:
 - Traveling Salesman Problem
 - Knapsack Problem
 - Vehicle/Arc Routing Problem
 - Timetabling problem
 - Map Colouring
 - •



Example: Vehicle Routing Problem with Time Windows

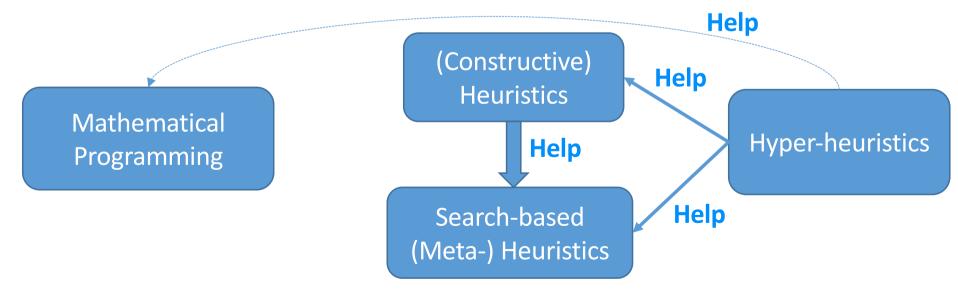
Methods for Combinatorial Optimisation

Exact methods

Mathematical programming

Approximated methods (heuristics)

- (Constructive) Heuristics
- Search-based Heuristics (Meta-heuristics)
- Hyper-heuristics



Mathematical Programming

- Guarantee Optimality
- Very mathematical demanding
- Can be very slow
- Not flexible in stochastic/dynamic environment
- Still need some **heuristics** (e.g. for branching)

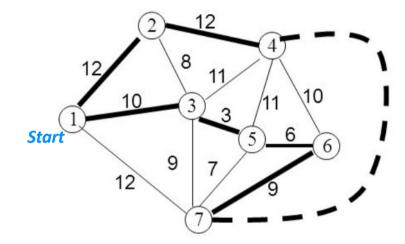
Max
$$2\kappa - \sum_{e \in E_R} h_e - \sum_{e \in E} \tilde{z}_e h_e$$

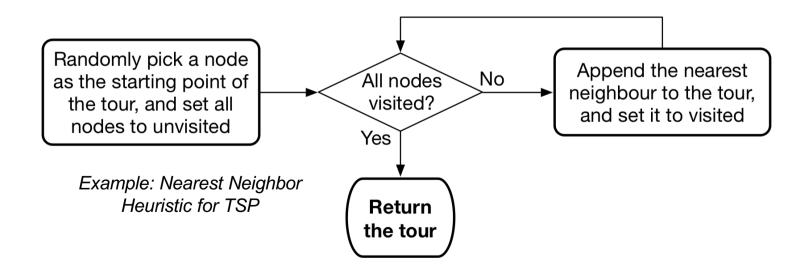
s.t. $h_e - s_i + s_j \ge 0 \quad \forall e = \{i,j\} \in E$
 $h_e + s_i - s_j \ge 0 \quad \forall e = \{i,j\} \in E$
 $-h_e + s_i + s_j \ge 0 \quad \forall e = \{i,j\} \in E$
 $s_i - f_e \ge 0 \quad \forall e = \{i,j\} \in E$
 $s_j - f_e \ge 0 \quad \forall e = \{i,j\} \in E$
 $s_j - f_e \ge 0 \quad \forall e = \{i,j\} \in E$
 $s_i + s_j - f_e \le 1 \quad \forall e = \{i,j\} \in E$
 $\sum_{e \in \delta(\{i\})} (h_e + f_e) - s_i \ge 0 \quad \forall i \in V$
 $h_e + f_e \le 1 \quad \forall e \in E$
 $\kappa = \sum_{e \in E} \frac{d_e(h_e + f_e)}{Q} + \gamma$
 $s_0 = 0$
 $h_e f_e \in \{0,1\} \quad \forall e \in E$
 $s_i \in [0,1] \quad \forall i \in V \setminus \{0\}$
 $\kappa \in \mathbb{Z}_0^+$

 $\gamma \in [0,1)$

(Constructive) Heuristics

- Incrementally construct a solution from scratch
 - Easy to understand and implement
 - Fast
 - Reasonably good solutions
 - Cannot guarantee optimality





Search-based Heuristics (Meta-heuristics)

- Iteratively improve one or more solutions
 - Produce high-quality solutions
 - Faster than mathematical programming
 - Can embed domain knowledge
 - Can combine with constructive heuristics (initial solutions)
 - Not flexible in stochastic/dynamic environment
 - Not scalable well to large problem size

Simulated Tabu Variable Memetic Annealing Neighborhood Algorithms Search Search **Guided Local Ant Colony** Genetic Search System **PSO** Algorithms

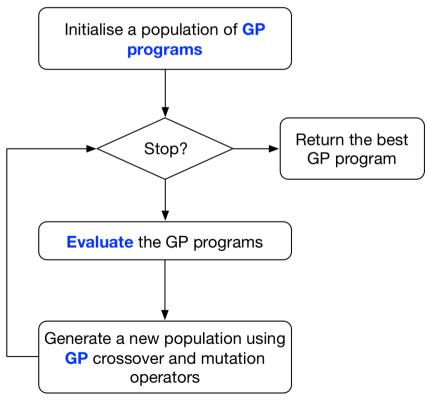
Hyper-heuristics

- Search for **heuristics** rather than solutions
 - Fast (Response immediately in dynamic environment)
 - Flexible (Solutions can be applied to a range of problem instances)
 - Scalable to large problems
 - Can discover new knowledge for problem solving
- A typical example: Genetic Programming Hyper-Heuristic (GPHH) for evolving dispatching rules for job shop scheduling

Branke, J., Nguyen, S., Pickardt, C.W. and Zhang, M., 2016. Automated design of production scheduling heuristics: a review. *IEEE Transactions on Evolutionary Computation*, 20(1), pp.110-124.

Genetic Programming

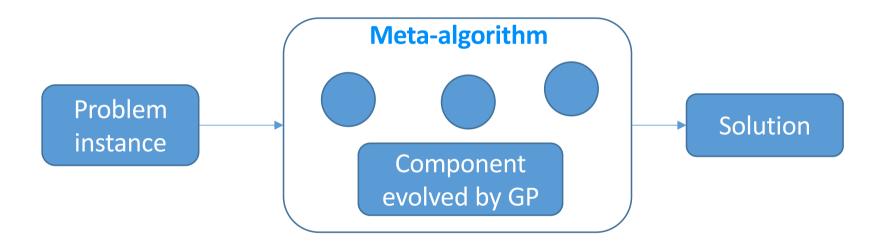
- Evolve a population of computer programs
- Crossover and mutation operators according to representation (e.g. tree, graph)



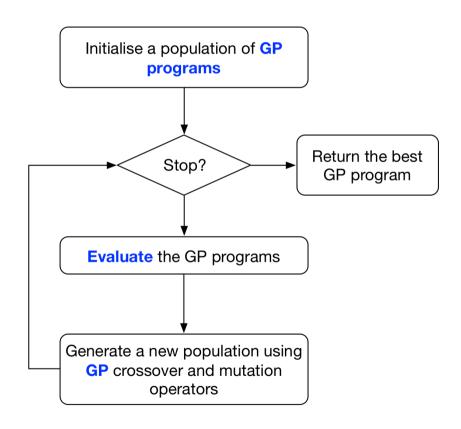
Genetic Programming as Hyper-Heuristic

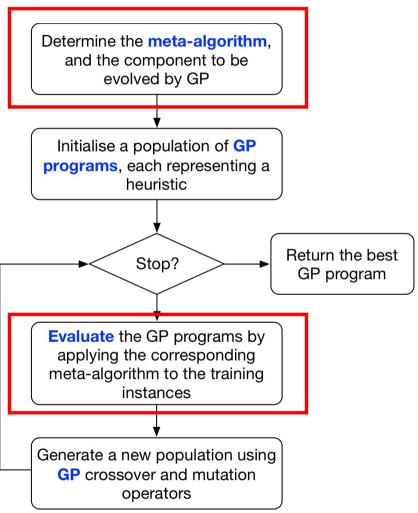
Meta-algorithms

• An algorithm to generate a solution given a problem instance



Genetic Programming as Hyper-Heuristic





Issues for GPHH

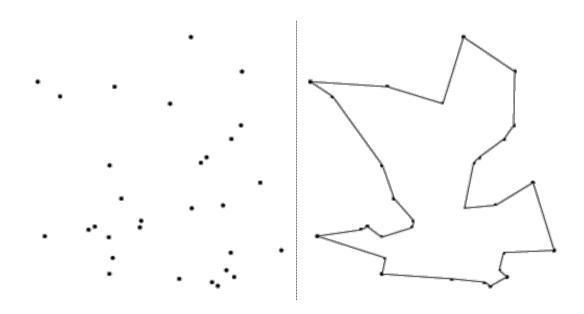
- How to represent a heuristic (GP program)?
 - Tree?
 - Graph?
 - Sequence?
- How to evaluate a heuristic?
 - Performance on a set of problem instances?
 - Generalisation? Performance on unseen instances?

Representation of Heuristics

Example: Constructive heuristic for TSP

Meta-algorithm

- Step 0: S = (), all nodes unvisited;
- Step 1: Select an unvisited node v^* based on some priority function, $S = (S, v^*)$;
- Step 2: If all nodes visited, return S, otherwise, go back to Step 1;

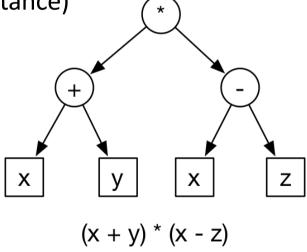


Representation of Heuristics

- Calculate the priority for all the unvisited node using the **priority** function $h(v; \Theta)$, then select the node with the highest priority
 - Nearest neighbour heuristic: $h(v; \Theta) = -d(v, S)$
- For evolving constructive heuristics for TSP using GP, one can represent the priority functions as syntax trees

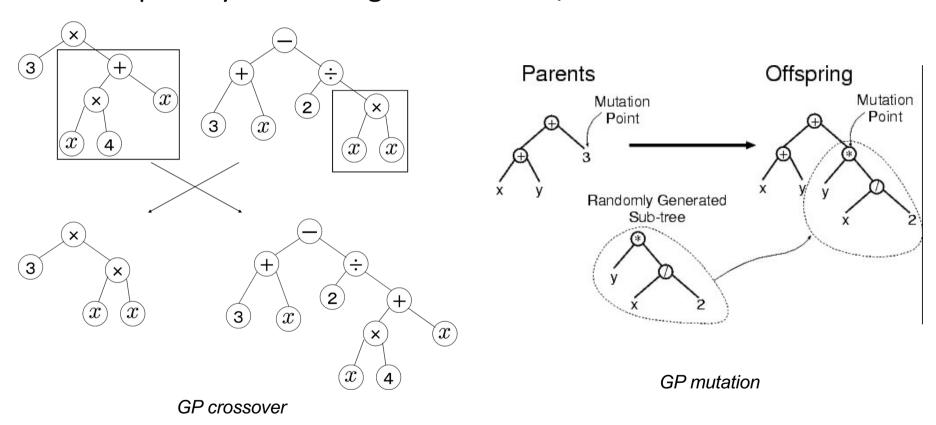
• Terminals: state features (e.g. location, distance)

• Functions: +, -, *, /, min, max, ...



Representation of Heuristics

• Evolve the priority trees using GP crossover/mutation



Poli, R., Langdon, W.B., McPhee, N.F. and Koza, J.R., 2008. A field guide to genetic programming.

Evaluation of Heuristics

- A heuristic π produces a solution given a problem instance
- Performance on an instance i: $perf(\pi, i)$ = objective value of the produced solution to the instance i
- Overall performance on a set of instances I: $perf(\pi, I) = mean of$ the normalised objective values of the produced solutions to each instance $i \in I$
 - Normalise by the lower bound
 - Normalise by the performance of reference heuristic/method
- But a heuristic perform well on the training instance(s) may not perform well on unseen instances (overfitting)
- Generalisation is an important issue (performance on unseen instances)

Evaluation of Heuristics

- Various strategies to improve generalisation
 - Use comprehensive training instances
 - Use small training set + change training set after each generation (similar to stochastic gradient descent/mini-batch in machine learning)
 - Regularisation: restrict the maximal depth of GP trees
 - Restrict the structure of GP trees (e.g. strongly-typed GP, grammar-based GP)

• ...

In This Talk...

- Evolve dispatching rules for job shop scheduling
- Evolve heuristics for arc routing problem
- Evolve heuristics for memetic algorithm in traveling thief problem

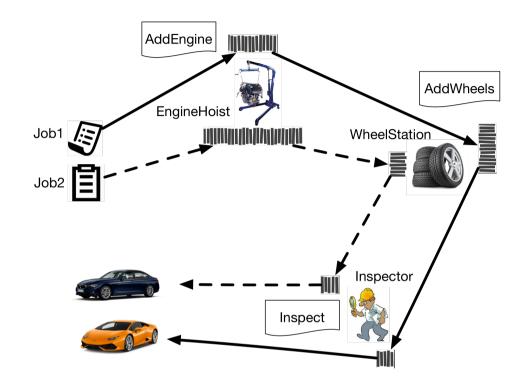
GPHH for Evolving
Dispatching Rules for
Job Shop Scheduling

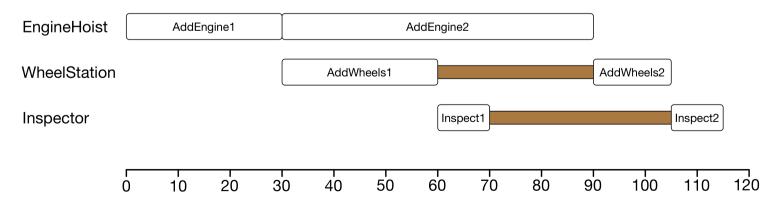
Job Shop Scheduling

- Process a set of jobs with a set of machines
- Each job has a sequence of operations, each processed by a certain machine
- Each job has arrival time, due date, weight, etc
- Each operation has a processing time
- Objective: minimise makespan/flowtime/tardiness
- Constraint
 - Each machine can process at most one operation at a time
 - An operation cannot start until its preceding operations have completed

Job Shop Scheduling

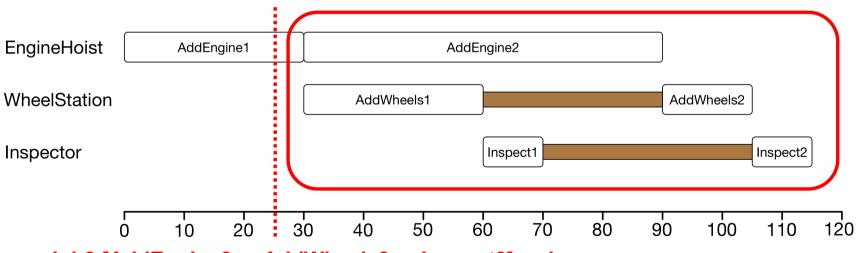
- Car manufacturing
- 3 machines (Engine Hoist, Wheel Station, Inspector)
- 2 jobs, each with 3 operations
 - 1) AddEngine
 - 2) AddWheel
 - 3) Inspect





Dynamic Job Shop Scheduling

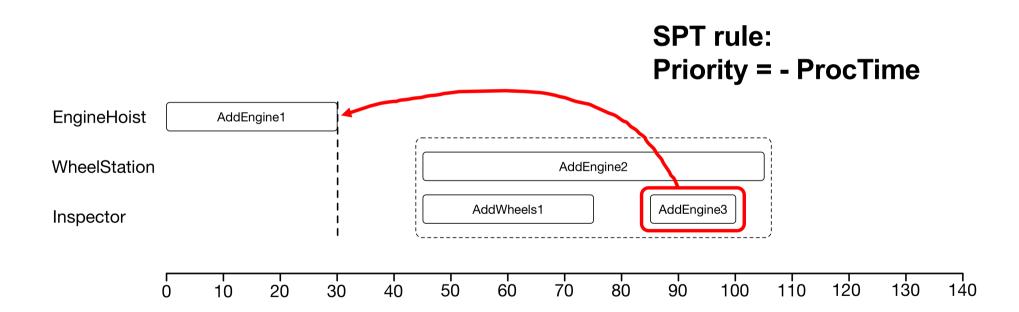
- Unpredicted events (e.g. new job arrivals) occur during the execution of the schedule
- Immediate response is needed
- Solution optimisation methods are usually too slow to respond effectively



Job3 [AddEngine3 -> AddWheels3 -> Inspect3] arrives

Dispatching Rule

- Whenever a machine becomes idle and its queue is not empty
 - Calculate the priority of the operations waiting in the queue
 - Select the most prior operation to process next

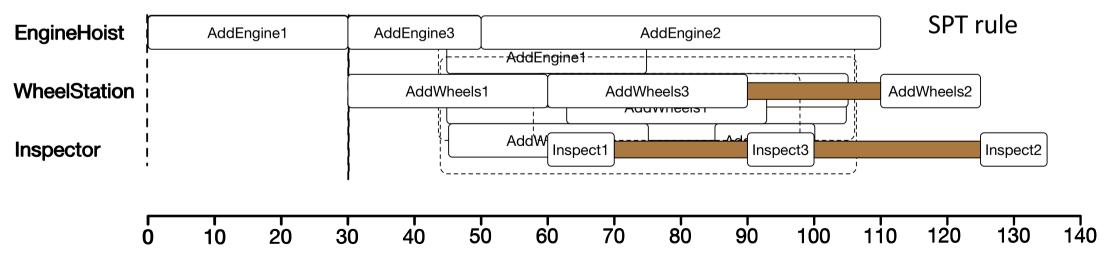


Dispatching Rule

- Many rules have been designed manually (FCFS, SPT, EDD, PT+WINQ, 2PT+WINQ+NPT, WATC, ...)
- Can handle dynamic JSS very well
 - Quick response
 - Good scalability (work well for huge problems)
 - Flexibility (can apply to a range of JSS instances)
- Manually designing effective dispatching rules is very challenging
 - Many interdependent factors (features) to consider
- Evolve dispatching rules using GPHH

Evolve Dispatching Rules by GPHH

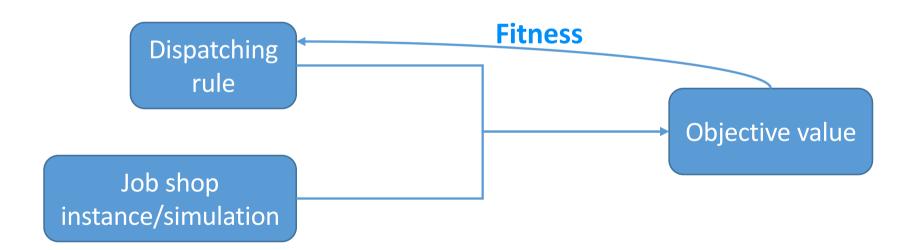
- Meta-algorithm: discrete event simulation
 - Start from time 0, empty schedule, initial jobs waiting in their machines
 - New jobs may arrive in real time (e.g. Poisson process)
 - As soon as a machine is idle and there are jobs waiting in its queue, select a
 job from its queue to be processed next using the dispatching rule
 - Stop if all jobs completed



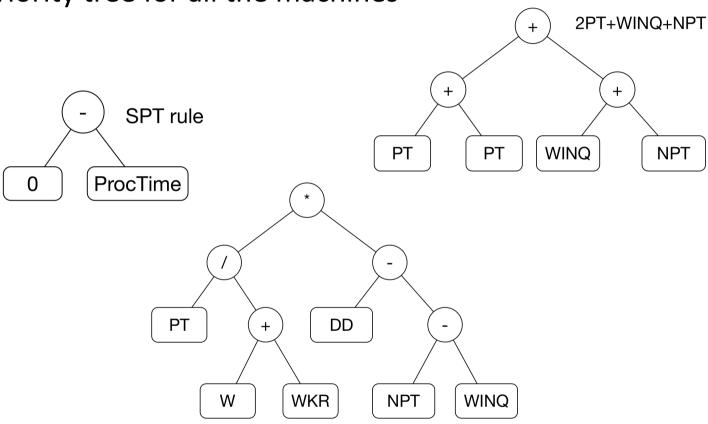
Evolve Dispatching Rules by GPHH

Objectives

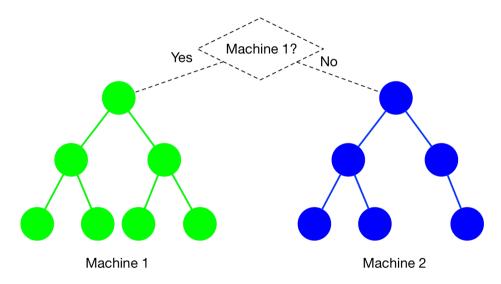
- Makespan: $\max C_i$
- Mean flowtime: $\frac{1}{N}\sum_{j=1}^{N}(C_j-a_j)$
- Mean weighted tardiness: $\frac{1}{N}\sum_{j=1}^{N}w_{j}T_{j}$, where $T_{j}=\max\{C_{j}-d_{j},0\}$



• Single priority tree for all the machines

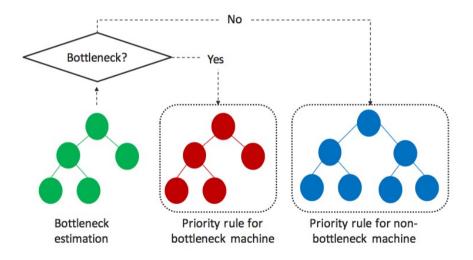


- Machine-specific priority trees
- Effective especially when machines have different scenarios
 - Unbalanced job shop
- Two machines with different utilisations



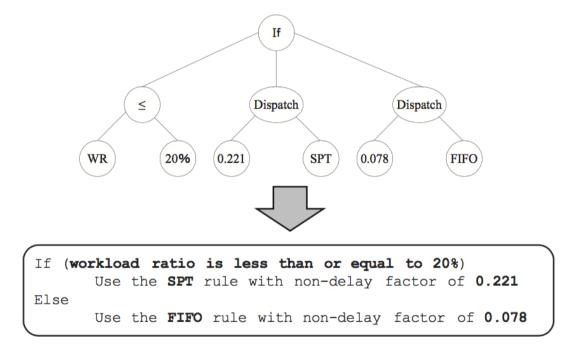
Hunt, R., Johnston, M. and Zhang, M., 2014, July. Evolving machine-specific dispatching rules for a two-machine job shop using genetic programming. In 2014 IEEE Congress on Evolutionary Computation (CEC) (pp. 618-625). IEEE.

- Machine-specific priority trees
- Effective especially when machines have different scenarios
 - Unbalanced job shop
- Bottleneck machines vs non-bottleneck machines



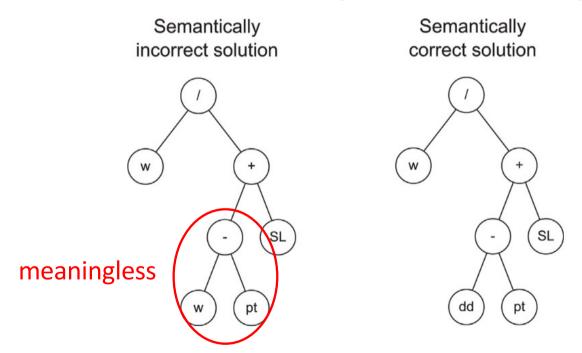
Jakobović, D. and Budin, L., 2006, April. Dynamic scheduling with genetic programming. In *European Conference on Genetic Programming* (pp. 73-84). Springer Berlin Heidelberg.

- Decision tree-like representation
 - Allow idle machines to wait some time even with non-empty queue



Nguyen, S., Zhang, M., Johnston, M. and Tan, K.C., 2013. A computational study of representations in genetic programming to evolve dispatching rules for the job shop scheduling problem. *IEEE Transactions on Evolutionary Computation*, *17*(5), pp.621-639.

- Dimensionality-Aware GP
 - Different attributes have different dimensions (units: time, count, weight, ...)
 - Keep semantic correctness with respect to dimensionality



Đurasević, M., Jakobović, D. and Knežević, K., 2016. Adaptive scheduling on unrelated machines with genetic programming. *Applied Soft Computing*, 48, pp.419-430.

- Many features: huge search space
- Some features are **redundant/irrelevant** (e.g. due date is irrelevant when minimising makespan)
- Select a subset of important features
- Feature selection is challenging as it depends on
 - Job shop scenario (utilisation level, due date factor, ...)
 - Objective (flowtime, tardiness, ...)
 - Complex interaction between features
- Learn the importance of features

- Ideally, we only need the features that contribute to the optimal individual
- However, the optimal individual is unknown

Approximation

- If a feature contributes to a **better** individual, then it is more likely to contribute to the optimal individual
- If a feature contributes to more individuals, then it is more likely to contribute to the optimal individual

- Use the number of appearances to measure the contribution of a feature to an individual
- Update the importance estimation during GP process

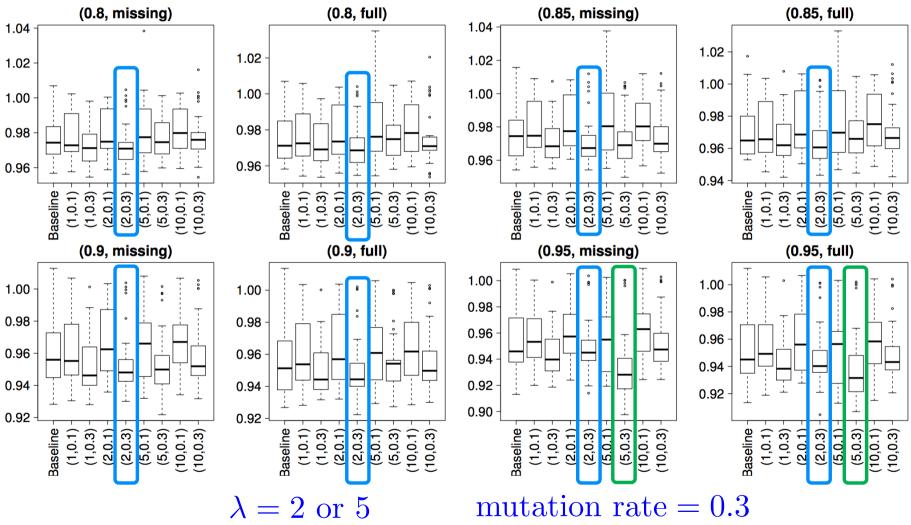
$$w_i \leftarrow w_i + \sum_{j=1}^{P} \frac{count_{ij}}{\sum_{k=1}^{n} count_{kj}} \times fitness_j$$

• During mutation, the **probability** of choosing a feature when generating the new sub-tree depends on its importance

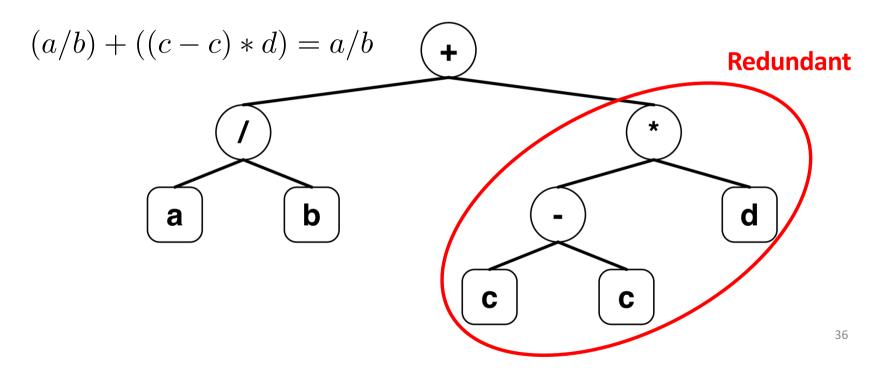
$$p_i = \frac{w_i^{\lambda}}{\sum_{k=1}^n w_k^{\lambda}}$$

Riley, M., Mei, Y. and Zhang, M., 2016, November. Improving job shop dispatching rules via terminal weighting and adaptive mutation in genetic programming. In *Evolutionary Computation (CEC)*, 2016 IEEE Congress on (pp. 3362-3369). IEEE.

- Experiments
 - 8 scenarios (4 utilisation levels \times 2 operation settings)
 - Utilisation: 0.8, 0.85, 0.9, 0.95
 - Ops:
 - Missing: uniform from 2 to the number of machines
 - Full: equal to the number of machines
 - λ values: 1, 2, 5, 10
 - 2 mutation rates: 0.1 and 0.3



Using number of appearances may be misleading



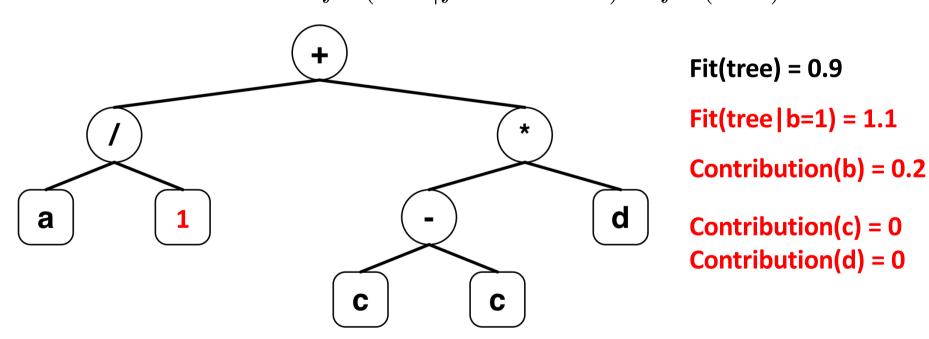
But feature c appears twice, which is more than a and b.

Feature Selection for GP Terminals

A new contribution measure

$$contribution(feature, tree) =$$

 $fit(tree|feature = 1) - fit(tree)$



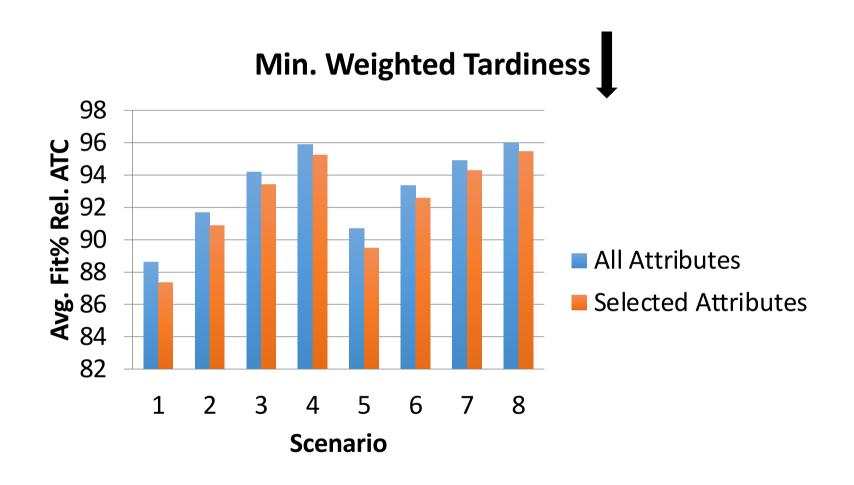
Yi Mei, Mengjie Zhang, Su Nguyen, "Feature Selection in Evolving Job Shop Dispatching Rules with Genetic Programming," Genetic and Evolutionary Computation Conference (GECCO), Denver, USA, 2016.

Feature Selection for GP Terminals

- Step 1: Conduct 30 pilot GP runs, collect 30 best individuals
- Step 2: Calculate contribution of each feature to each individual
- Step 3: Select a feature if it contributes to more than 15 individuals

	<u> </u>	
Notation	Description	
NOW	The current time.	in weighted tardiness
PT	Processing time of the operation.	iii weigiited taruiiless
IPT	Inverse of the processing time.	
NOPT	Processing time of the next operation.	
ORT	Ready time of the operation.	
MRT	Ready/Idle time of the machine.	
NMRT	Ready time of the next machine.	
WIQ	Work in the current queue.	
WINQ	Work in the next queue.	
NOIQ	Number of operations in the current queue	•
NOINQ	Number of operations in next queue.	
WKR	Work remaining (including the current operati	on).
NOR	Number of operations remaining.	
FDD	Flow due date of the operation.	
$_{ m DD}$	Due date of the job.	
W	Weight of the job.	38

Feature Selection for GP Terminals

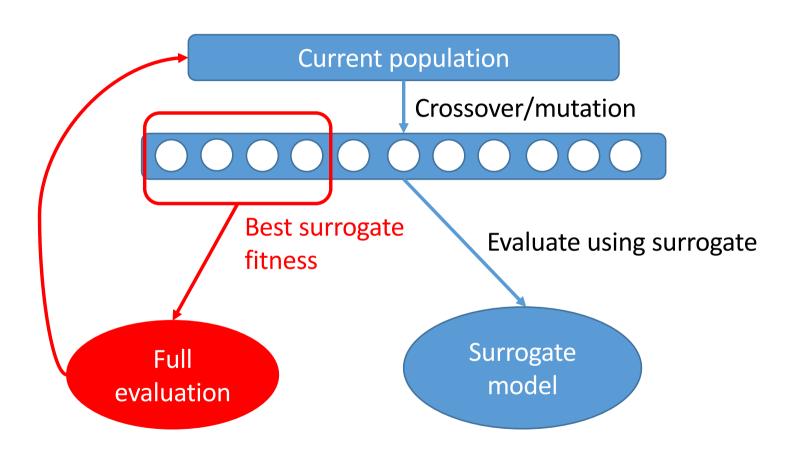


Evaluation Model

Standard

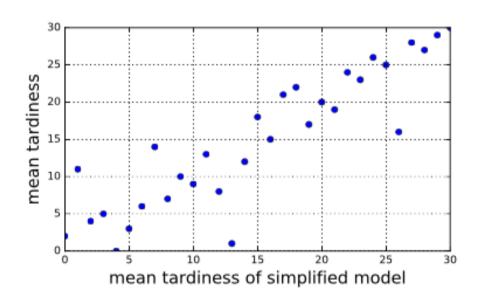
- A set of static instances (normalised by lower bound/reference rule)
- Dynamic discrete event simulation(s)
 - 10 machines, 2500 jobs, 2~10 operations per job
 - 500 warm-up jobs for steady-state performance
 - Different utilisation levels (0.85, 0.9, 0.95) and due date factors (3, 4, 5)
- Change the random seed of the simulation(s) at each generation
 - Much better generalisation
 - Much faster (only one replication per generation)

Hildebrandt, T., Heger, J. and Scholz-Reiter, B., 2010, July. Towards improved dispatching rules for complex shop floor scenarios: a genetic programming approach. In *Proceedings of the 12th annual conference on Genetic and evolutionary computation* (pp. 257-264). ACM.



• Smaller job shop simulation

	Original	Surrogate
No. Machines	10	5
No. Jobs	5000	500
No. Warmup Jobs	500	100
Min Ops	2	2
Max Ops	14	7



Nguyen, S., Zhang, M. and Tan, K.C., Surrogate-Assisted Genetic Programming With Simplified Models for Automated Design of Dispatching Rules. IEEE Transactions on Cybernetics, in press, DOI 10.1109/TCYB.2016.2562674.

Phenotypic characterisation

- A set of decision situations and a reference rule
- For each decision situation, measure the difference between the reference rule and the characterised rule
- Characterised by a decision vector

decision	attri	bute				
situation	s ₁	s_2	s_3	reference rule	other rule	vector d
1 1	3 7	4 6	8 15	2	2 1	2
2 2 2	23 8 6	17 9 4	1 3 6	2 3 1	2 1 3	3
:		:		:	:	:
k k	7 4	3 8	9	2	2 1	1

Hildebrandt, T. and Branke, J., 2015. On using surrogates with genetic programming. *Evolutionary computation*, 23(3), pp.343-367.

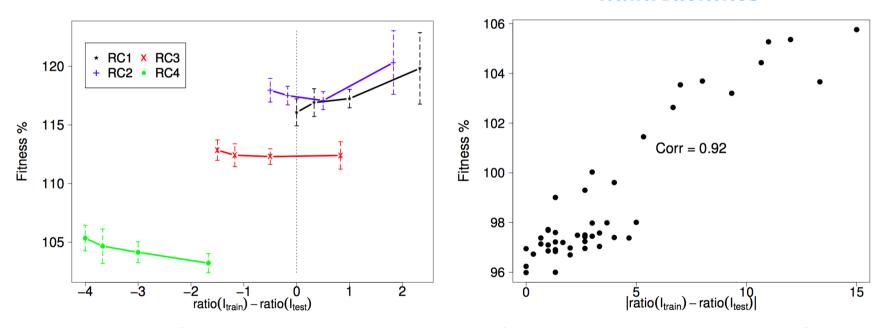
- If two rules have similar phenotypic characterisation, i.e. decision vectors, then they tend to have similar fitness values
- A <decision vector, fitness> database (the fully evaluated individuals in the last 2 generations)

	d_1	d_2	 d_k	fitness
rule₁:	2	3	 1	1456
rule ₁ : rule ₂ :	1	2	 2	1456 1123
:		÷		:
rule _m :	1	3	 1	1293

 Nearest neighbour regression – set the approximated fitness to the fitness of the closest rule in the database

Surrogate vs Reusability

- In static case, we aim to train dispatching rules using small instances (surrogate), which can be reused on large instances
- Such reusability strongly relates to $ratio = \frac{numJobs}{numMachines}$

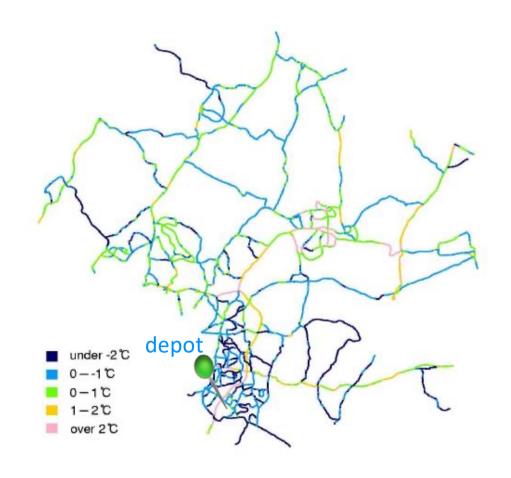


Yi Mei, Mengjie Zhang, "A Comprehensive Analysis on Reusability of GP-Evolved Job Shop Dispatching Rules," *IEEE World Congress in Computational Intelligence (WCCI)*, Vancouver, Canada, 2016.

GPHH for Evolving Heuristics for Arc Routing Problem

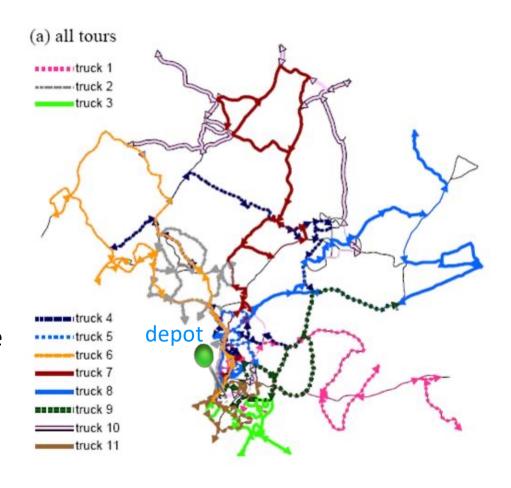
Arc Routing Problem

- A Graph
 - A set of arcs to be served (tasks)
 - A special node (depot)
- Arc
 - Demand
 - Serving cost
 - Deadheading cost
- A fleet of vehicles
 - Capacity



Arc Routing Problem

- A solution
 - A set of routes to serve the tasks
- Objective
 - Minimize the total cost
- Constraints
 - Each task is served exactly once
 - Each vehicle starts and ends at the depot
 - The total demand served by each vehicle cannot exceed its capacity



Developmental CARP Solving

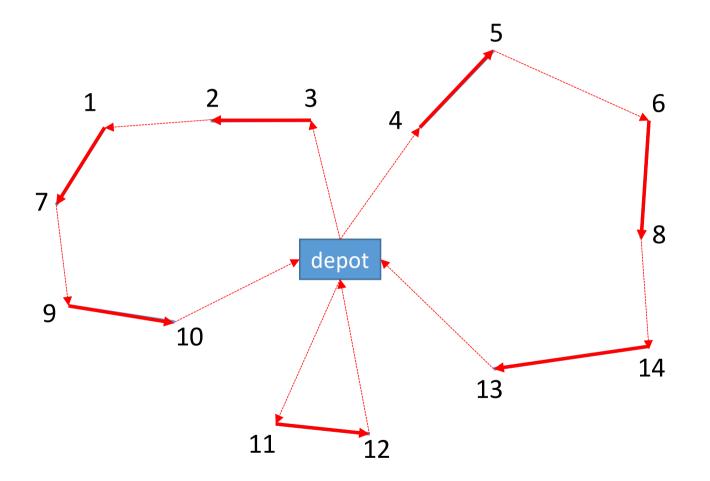
A single vehicle, but can go back to refill

Meta-algorithm

- **Step 0**: A vehicle at the depot, all tasks unserved;
- **Step 1**: Select an unserved task by the **heuristic function**;
- **Step 2**: If the vehicle can serve the task without violating the capacity constraint, then go; otherwise go back to the depot to refill;
- **Step 3**: If all the tasks have been served, then go back to depot and stop; otherwise go to Step 1;

Weise, T., Devert, A. and Tang, K., 2012, July. A developmental solution to (dynamic) capacitated arc routing problems using genetic programming. In *Proceedings of the 14th annual conference on Genetic and evolutionary computation* (pp. 831-838). ACM.

Developmental CARP Solving



Decisions

Go to 3, serve <3,2>

Go to 1, serve <1,7>

Go to 9, serve <9,10>

Go back to depot

Go to 4, serve <4,5>

Go to 6, serve <6,8>

Go to 14, serve <14,13>

Go back to depot

Go to 11, serve <11,12>

Go back to depot

Evolve Heuristic Function to Make Decisions

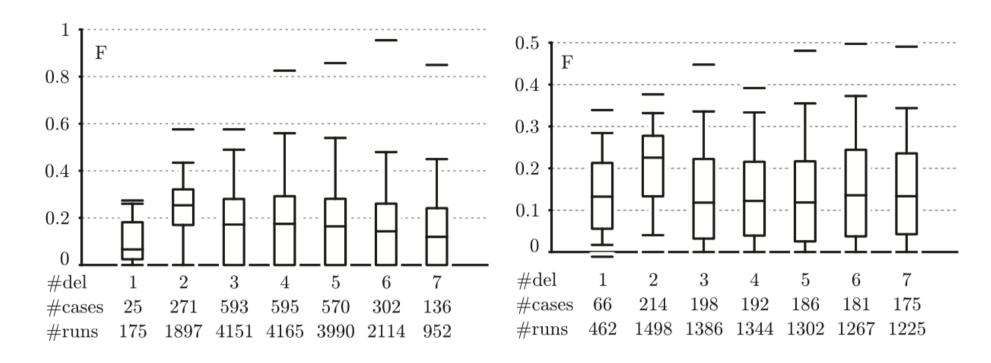
Standard GP

- A single tree to calculate heuristic value
- Select the task with the lowest heuristic value

Terminal	Description		
Demand(e)	Demand of the task e		
Load	Remaining load of the vehicle / capacity		
Cost(e)	Cost of the task e		
DepotCost(e)	Cost to go back to depot from task e		
Satisfied	Fraction of satisfied (served) tasks		
Last(e)	Heuristic value calculated in the last round		

+, -, *, /, max, exp, sin, angle

Results



• Outperform existing heuristics in uncertain environment

Open Issues

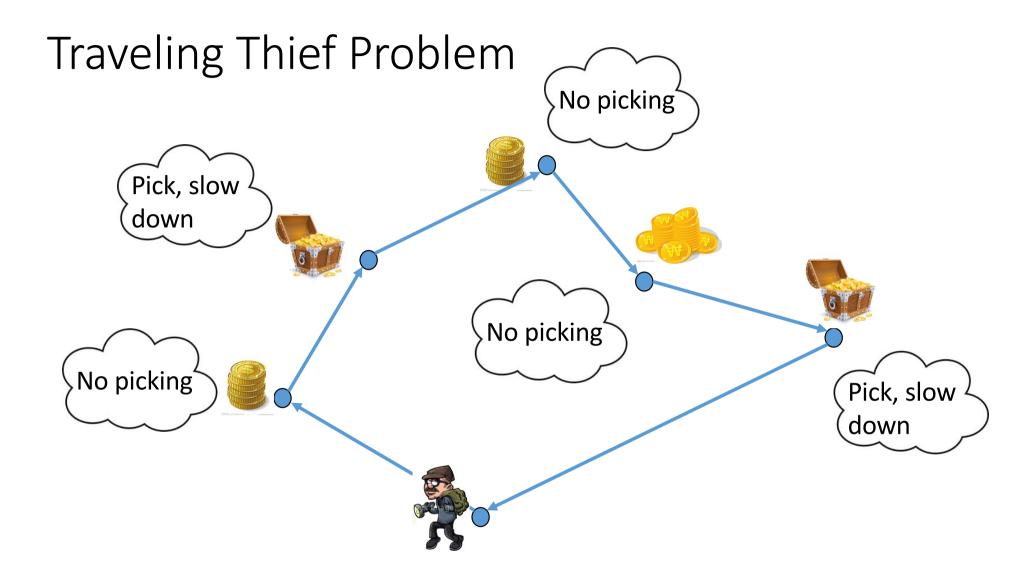
- Generalisation
- Dynamic problems (new tasks arrive in real time)
- Multiple vehicles serving simultaneously
- Better meta-algorithms
- Interpretability

GPHH for Evolving Heuristics for Memetic Algorithm in TTP

Traveling Thief Problem

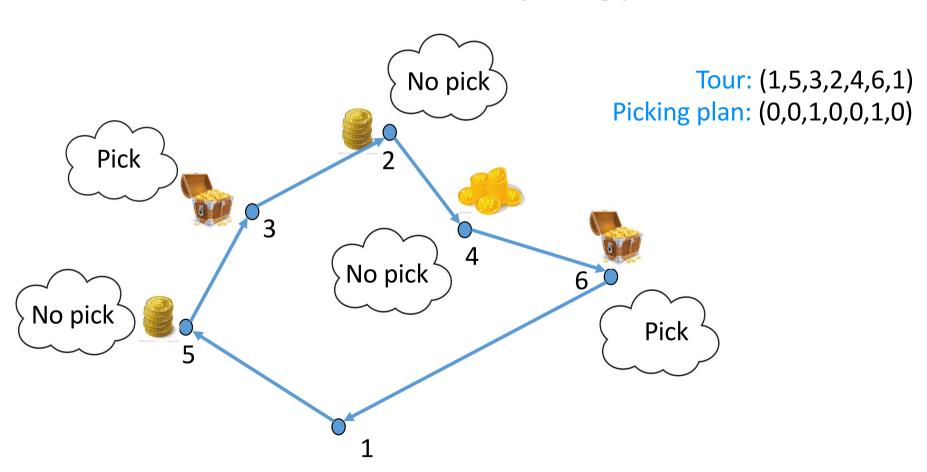
- A new benchmark problem for studying interdependent components
- A combination of TSP and KP
- A set of cities
- Each city has an item
- Each item has a value and a weight
- A thief with capacity and a speed depending on weight carried
- Visit all the cities and collect some items to maximise profit

$$\sum_{i \in selected} c_i - \alpha \cdot T$$

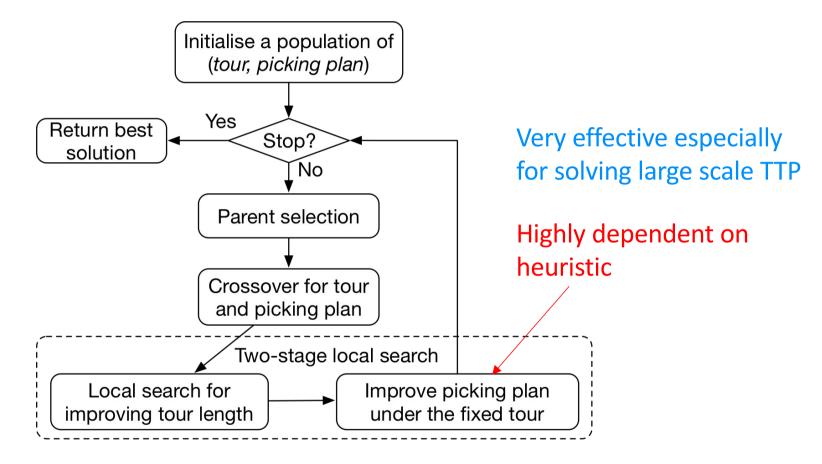


Traveling Thief Problem

A solution contains a TSP tour and a picking plan



Memetic Algorithm for TTP



Mei, Y., Li, X. and Yao, X., 2014, December. Improving efficiency of heuristics for the large scale traveling thief problem. In *Asia-Pacific Conference on Simulated Evolution and Learning* (pp. 631-643). Springer International Publishing.

- Different from conventional knapsack heuristics
- The efficiency of an item depends on
 - value weight
 - Distance from where it is to the starting city (not to slow down too early)





















$$c=3$$
 $c=1$

$$w = 1 \qquad w = 1 \qquad w = 1$$

$$d = 10$$
 $d = 8$ $d = 6$

$$c = 1$$

$$W-1$$

$$d = 8$$

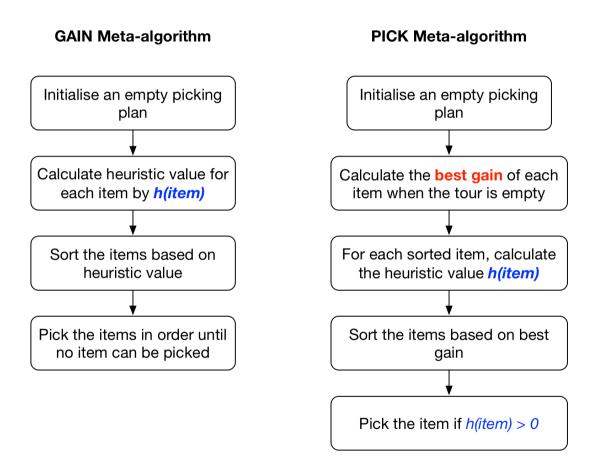
$$c=1$$

$$d = 6$$

$$c = 2 \qquad c = 3$$
$$w = 1 \qquad w = 1$$

$$d = 2$$
 $d = 1$

- A very sophisticated heuristic
 - Step 0: All items not picked, current load of the tour is zero;
 - Step 1: For each item, calculate the best gain when the tour is empty;
 - Step 2: Sort the item in the decreasing order of the best gain;
 - Step 3: For each sorted item, if feasible and expected gain under the current load of the tour is positive, then pick the item and update the current load of the tour
 - Step 4: If all sorted item is scanned, stop; otherwise go to the next sorted item;
- Complex calculation formulas for the best gain and expected gain
- Evolve using GP

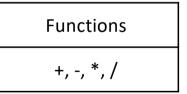


Mei, Y., Li, X., Salim, F. and Yao, X., 2015, May. Heuristic evolution with genetic programming for traveling thief problem. In 2015 IEEE Congress on Evolutionary Computation (CEC) (pp. 2753-2760). IEEE.

Evaluation Model

- Three small TTP instances
- Run MA with the heuristic function once, and get the best solution
- Fitness of the heuristic = the fitness of this best solution

Terminal	Description
profit	Profit of the item
weight	Weight of the item
bdist	Distance of the location of the item to the end of the tour
Q	The capacity of the knapsack
$egin{array}{c} Q \ L \end{array}$	The total length of the tour
R	The rent ratio of the knapsack
u	The coefficient defined by Eq. (2)
$v_{ m max}$	The maximal speed
W	The total weight of the items selected so far



• Very similar performance as the manually designed heuristic

Name	n	m	TSMA	TSMA-GAIN	TSMA-PICK
brd14051	14051	140500	2.66e+7(2.07e+5)	2.66e+7(2.59e+5)	2.66e+7(3.41e+5)
d15112	15112	151110	2.85e+7(5.35e+5)	2.83e+7(3.40e+5)	2.84e+7(4.75e+5)
d18512	18512	185110	3.07e+7(2.73e+5)	3.07e+7(3.47e+5)	3.06e+7(4.95e+5)
pla33810	33810	338090	6.34e+7(4.59e+5)	6.32e+7(6.87e+5)	6.33e+7(5.11e+5)
rl11849	11849	118480	1.97e+7(8.29e+4)	1.97e+7(9.06e+4)	1.97e+7(1.04e+5)
usa13509	13509	135080	2.92e+7(2.60e+5)	2.92e+7(2.55e+5)	2.92e+7(1.70e+5)

Conclusion

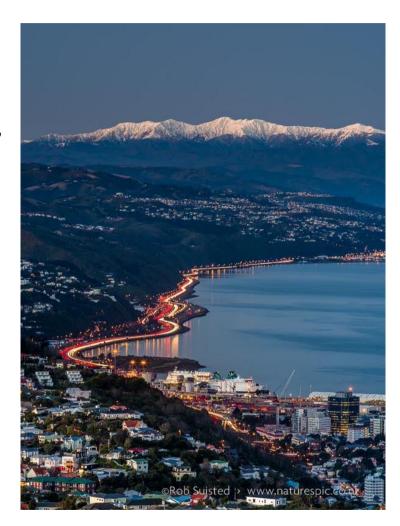
- Genetic Programming has been successfully used as a hyper-heuristic for automatically designing heuristics
- Very useful in combinatorial optimisation, where heuristics are usually needed for decision making
- Especially powerful in dynamic environment, in which immediate response is needed
- Many open issues to be addressed
 - Representation
 - Evaluation model
 - Generalisation
 - Interpretability
 - •

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- Evolutionary Image Analysis and Pattern Recognition
- Evolutionary Machine Learning and Transfer Learning
- Genetic Programming, PSO, Learning Classifier Systems

More details

https://ecs.victoria.ac.nz/Groups/ECRG/ResearchAreas#Areas

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- Research experience/publications in EC, combinatorial optimisation, ...
- Strong programming skills in Java, Python, R, ...